Non Destructive Testing: Characterization of a crack with the measurement of an electric resistance

2D Textbook Case Summary

Program	Dimension	Physics	Application	Work area
Flux	2D	Electric	Kinetic	NDT

Characterization of the geometry of a crack with the measurement of an electric resistance. This example shows how to use one of the existing Non Destructive Testing methods (NDT by conduction or by current injection).

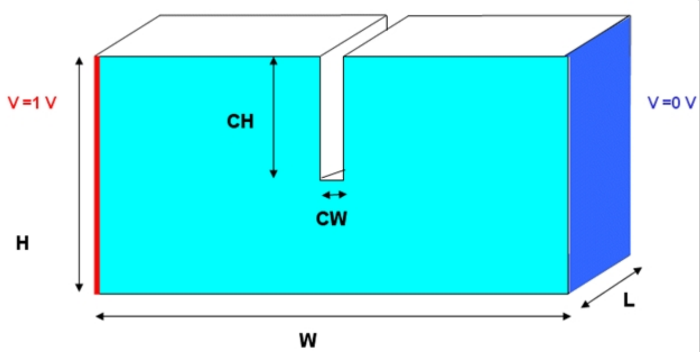
Objective: Computation of the electric resistance of a cracked component. The parameters the user can change are:

- crack's width (CW)
- crack's height (CH)
- resistivity of the component's material (RHO)

Theoretical reminders

Analytical computation of elementary electric resistances of components shaped as parallelepipeds or prisms.

$$dR = \frac{\rho \times dl}{S(h)}$$

Illustration	Main characteristics
	• Rated potential difference V = 1 V • Rated component dimensions: W x H x L = 100 mm x 50 mm x 40 mm • Rated material resistivity (copper) RHO = 1.7 E-8 Ω.m • Rated crack's width: CW = 5 mm • Rated crack's height: CH = 45 mm

Results

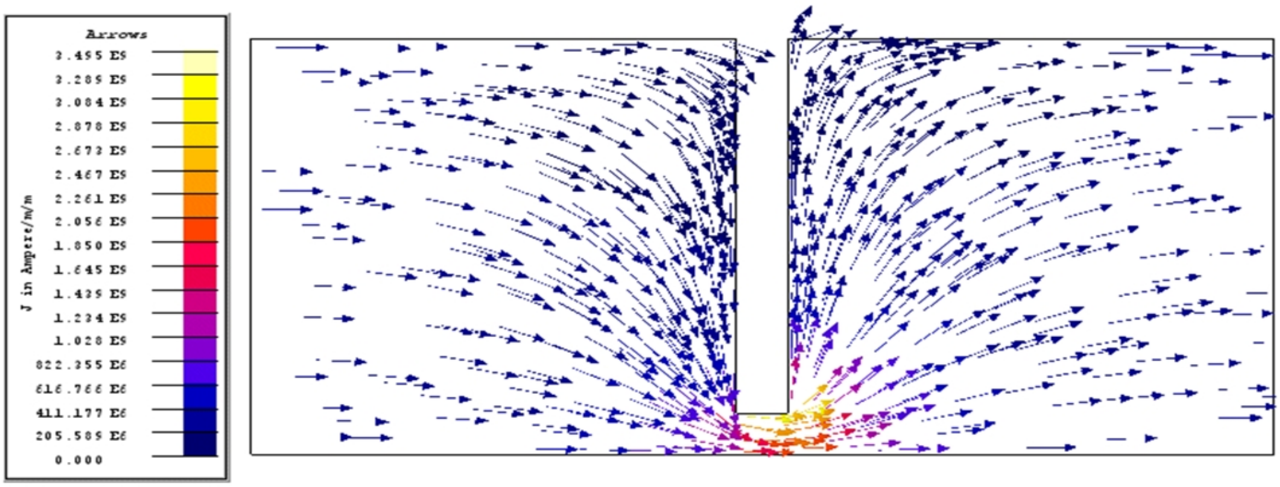


Figure 1: Flow of current at rated working point

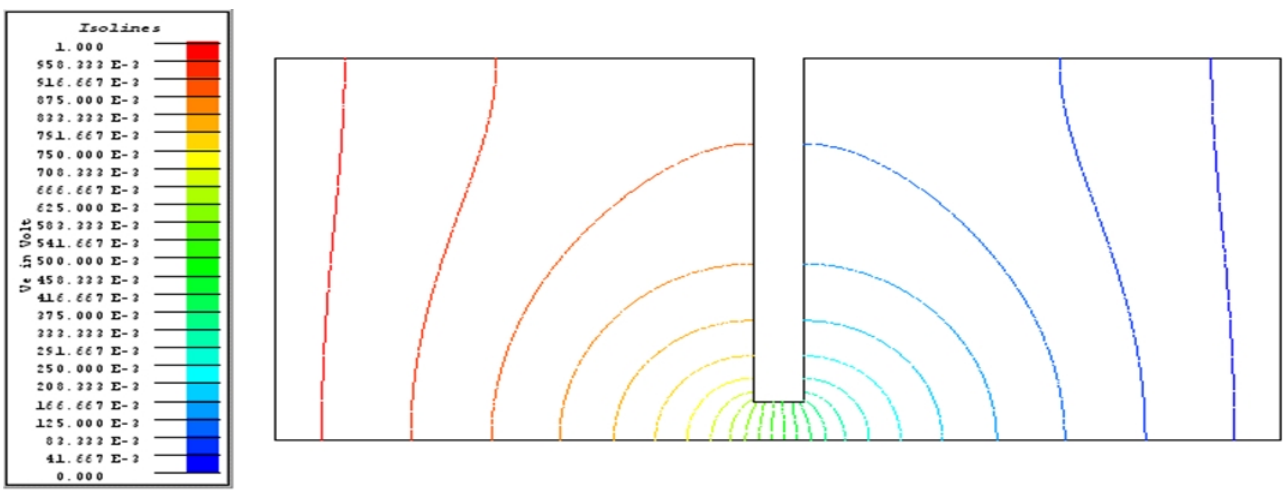


Figure 2: Equipotential lines at rated working point

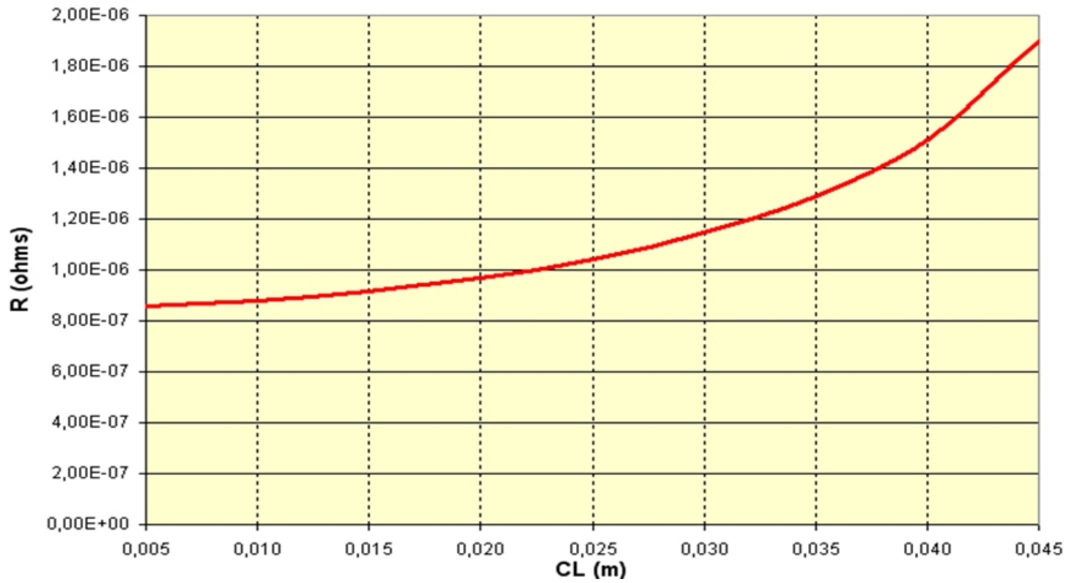


Figure 3: Electric resistance's (R) evolution as a function of the crack's height (CH) (the other parameters are rated)

To go further:

- Similar study, but with a stiff current density
- Similar analysis for more complex 3D type devices etc.

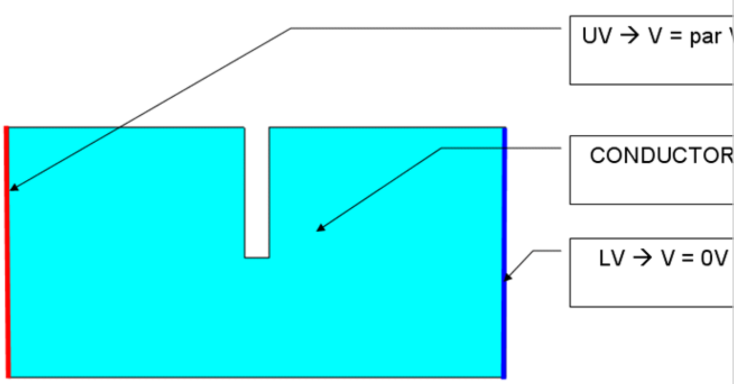
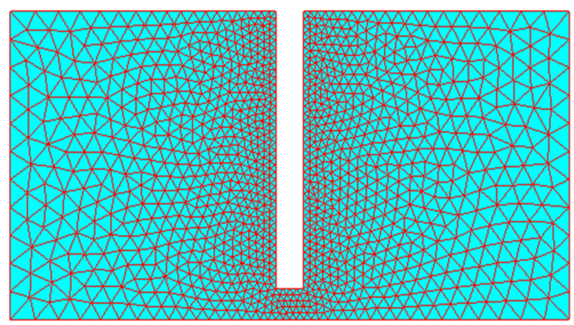
Model in Flux

Domain

Dimension	2D	Depth	L
Length unit.	mm	Angle unit.	degrees

Application	Electro kinetic

Geometry / Mesh

Full model in the FLUX environment	Mesh
	

Mesh	2nd order type	Number of nodes	4633

Input Parameters

Name	Type	Description	Rated value
L	Geometrical	Material length	40 mm
W	Geometrical	Material width	100 mm
H	Geometrical	Material height	50 mm
CH	Geometrical	Crack height	45 mm
CW	Geometrical	Crack width	5 mm
RHO	Physical	Material resistivity	1.7 E^{-8}W.m
V	Physical	Potential difference	1 V

Material Base

NAME	MATERIAL
B(H) model	-
Magnetic property	-
J(H) model	Isotropic resistivity
Electrical property	RHO
D(E) model	-
Dielectric property	-
K(T) model	-
K(T) characteristics	-
RCP(T) model	-
RCP(T) characteristics	-

Regions

NAME	COMPONENT	UV	LV
Nature	Surface region	Line region	Line region
Type	Conductive region	Stiff electric potential	Stiff electric potential
Material	MATERIAL	-	-
Mechanical Set	-	-	-
Corresponding circuit component	-	-	-
Electrical characteristics	-	V	0
Current source	-	-	-
Thermal characteristics	-	-	-
Possible thermal source	-	-	-

Solving process options

Type of linear system solver	Automatically chosen	Parameters	Automatically defined

<table>
<tr><td rowspan="2">Type of non-linear system solver</td><td rowspan="2">Newton Raphson</td><td>Precision</td><td>0.0001</td><td>Nb iterations</td><td>100</td></tr>
<tr><td colspan="2">Method for computing the relaxation factor</td><td colspan="2">Automatically chosen</td></tr>
</table>

Solving

Scenario	Name of parameter	Controllable parameter	Variation method	Interval definition	Step selection
SCENARIO	CH	Geometrical	Step value	5 mm to 45 mm	5 mm

Annex

Analytical computation of the resistance

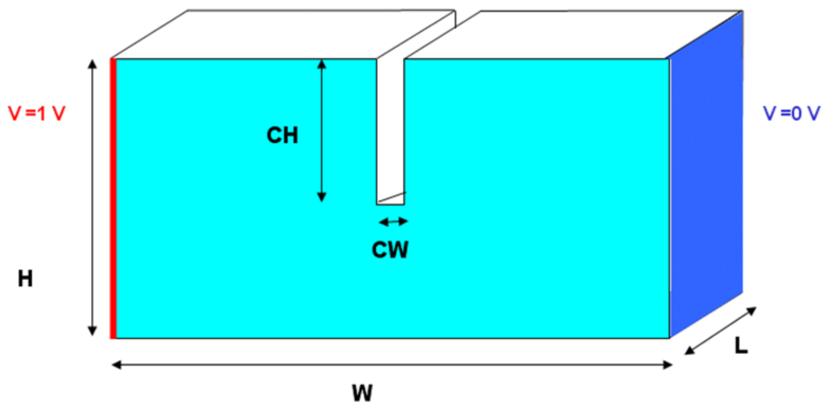
Maxwell equation: E=-gradV

Ohm's Law: $J = \sigma E$

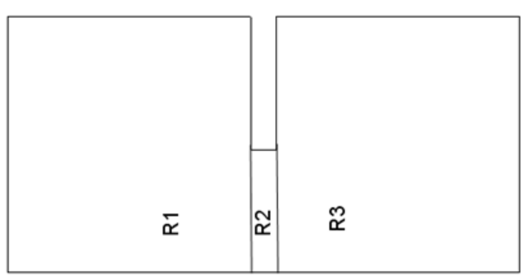
Resistance computation: $dR = \rho \times \frac{dh}{S(h)}$

General remarks

The analytical method consists of calculating the values of elementary electric resistances of components described as parallelepipeds (model 1) or parallelepipeds + prisms (model 2). The elementary resistances are then connected in series and a global equivalent resistance can be calculated.



Model 1 :

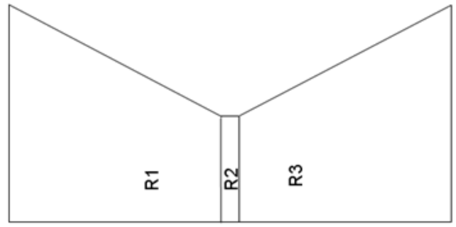


With: R=R1+R2+R3

$$R1 = R3 = \rho \times \frac{W\text{-}CW}{2 \times H \times L}$$

$$R2 = \rho \times \frac{CW}{(H\text{-}CH) \times L}$$

Model 2



With: R=R1+R2+R3

$$R1 = R3 = \rho \times \frac{CW}{2 \times L \times (H - CH)} \quad R2 = \rho \times \frac{W - CW}{L \times CH} \times \ln\left(\frac{H}{H - CH}\right)$$

Notations and symbols

Name	Description	Unit
R	Material resistance	Ω
ρ	Material resistivity	Ω.m
H	Material height	m
L	Material length	m
W	Material width	m
CH	Crack height	m
CW	Crack width	m

Numerical applications

R computation

Let's calculate the value of the electric resistance with different methods when component parameters are the following:

- Potential difference V = 1 V
- Component dimensions: W x H x L = 100 mm x 50 mm x 40 mm
- Material resistivity (copper): RHO = 1.7 E^{-8}Ω.m
- Crack width : CW = 5 mm
- Crack height : CH = 45 mm

Computation with method 1

$$R1 = R3 = \rho \times \frac{W - CW}{2 \times H \times L} = 1.7 \times 10^{-8} \times \frac{(100 - 5) \times 10^{-3}}{2 \times 50 \times 40 \times 10^{-6}} = 403 n\Omega$$

$$R2 = \rho \times \frac{CW}{(H - CH) \times L} = 1.7 \times 10^{-8} \times \frac{5 \times 10^{-3}}{(50 - 45) \times 40 \times 10^{-6}} = 425 n\Omega$$

$$R = R1 + R2 + R3 = 1.23 \mu\Omega$$

Computation with method 2

$$R1 = R3 = \rho \times \frac{CW}{2 \times L \times (H - CH)} = 1.7 \times 10^{-8} \times \frac{5 \times 10^{-3}}{2 \times 40 \times (50 - 45) \times 10^{-6}} = 212 n\Omega$$

$$R2 = \rho \times \frac{W - CW}{L \times CH} \times ln\left(\frac{H}{H - CH}\right) = 1.7 \times 10^{-8} \times \frac{95 \times 10^{-3}}{40 \times 45 \times 10^{-6}} \times ln\left(\frac{50}{5}\right) = 2.06 \mu\Omega$$

$$R = R1 + R2 + R3 = 2.46 \mu\Omega$$

Result obtained with Flux

At this working point, Flux calculates a resistance of $R = 2.46 \mu\Omega$